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## EXPLICIT CONDITIONS FOR THE EXISTENCE OF OSCILLATORY SOLUTIONS IN GENERALIZED LIÉNARD SYSTEM

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**Abstract.** The existence of oscillatory solutions, is one of important subjects in qualitative theory of differential equations and dynamical systems. In this work, some implicit necessary and sufficient conditions and some explicit conditions will be presented for solutions of generalized Liénard type system to be oscillatory or nonoscillatory.

Keywords. Oscillation, Liénard system, Dynamical System, Planar System, ODE.

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## 1 Introduction

Consider the planar system of differential equations

$$\frac{dx}{dt} = \frac{1}{a(x)} \left[ h(k(y) - F(x)) \right]$$

$$\frac{dy}{dt} = -a(x)g(x),$$
(1.1)

which is an extension of classical Liénard system. In this paper, suppose that the following assumptions hold.

(C<sub>1</sub>) F(x), a(x) and g(x) are continuous on  $\mathbb{R}$  with F(0) = 0, xg(x) > 0for  $x \neq 0$  and a(x) > 0 for  $x \in \mathbb{R}$ , h(u) and k(y) are continuously differentiable and strictly increasing with k(0) = h(0) = 0 and yk(y) > 0 for  $y \neq 0$  and  $h(\pm \infty) = \pm \infty$ .

Under these assumptions, the origin is the unique equilibrium for system (1.1).

In [10], the authors proved the following proposition about the existence and uniqueness of solution for initial value problem corresponding to system (1.1).