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EXISTENCE OF POSITIVE SOLUTIONS OF A SINGULAR FRACTIONAL BOUNDARY VALUE PROBLEM

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Abstract. Let $n \in \mathbb{N}$, $n \geq 3$. For $\alpha \in (n-1, n]$, we obtain the existence of positive solutions of the singular fractional differential equation $D^{\alpha}_{0+}u + f(t, u) = 0$, $0 < t < 1$, satisfying the boundary conditions $u^{(i)}(0) = 0$, $i = 0, \dots, n-2$, $D^{\beta}_{0+}u(1) = 0$, where $\beta \in [1, n-1]$. Here f is singular at $u = 0$, $t = 0$, $t = 1$, and possibly at $u = \infty$, and is decreasing in its space variable u. The main tool in the analysis is the Gatica, Oliker, and Waltman fixed point theorem.

Keywords. Fractional differential equation, fixed point, singular, positive solution.

AMS (MOS) subject classification: 26A33, 34A08, 34B16.

1. Introduction

Let $n \in \mathbb{N}$, $n \geq 3$. For $\alpha \in (n-1, n]$, consider the singular fractional differential equation

$$D^{\alpha}_{0+}u + f(t, u) = 0, \quad 0 < t < 1, \tag{1.1}$$

satisfying the boundary conditions

$$u^{(i)}(0) = 0,\ i = 0, \dots, n-2, \quad D^{\beta}_{0+}u(1) = 0, \tag{1.2}$$

where $\beta \in [1, n-1]$, D^{α}_{0+}, D^{β}_{0+} are the Riemann-Liouville fractional derivatives of order α and β, respectively, and $f(t, u)$ is singular at $t = 0$, $t = 1$, $u = 0$, and possibly at $u = \infty$. Here, we assume

- (H1) $f(t, u) : (0, 1) \times (0, \infty) \to (0, \infty)$ is continuous;
- (H2) $f(t, u)$ is decreasing in u for each t; and
- (H3) $\lim_{u \to 0^+} f(t, u) = \infty$ and $\lim_{u \to \infty} f(t, u) = 0$, uniformly on compact subsets of $(0, 1)$.

Singular fractional boundary value problems have been studied at length in recent years. In [1], using Krasnosel'skii's fixed point theorem [8], Agarwal, O'Regan, and Staněk obtained the existence of at least one positive solution