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EXPONENTIAL STABILITY AND BOUNDEDNESS OF SOLUTIONS FOR PERIODIC DISCRETE CAUCHY PROBLEMS

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Abstract. We prove some characterizations for exponential stability of a periodic semigroup of bounded linear operators acting on a Banach space in terms of discrete evolution semigroups acting on a special space of almost periodic sequences.

Keywords. Discrete evolution semigroup, Discrete linear systems, Uniform exponential stability, Periodic sequences, Almost periodic sequences.

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1 Introduction

If we denote by A a bounded linear operator acting on a complex Banach space X, a well-known theorem due to M. G. Krein, [13, 19] shows that the differential system $x'(t) = Ax(t)$ is uniformly exponentially stable if and only if for each real paramater $\mu \in \mathbb{R}$ and all $y_0 \in X$, the solution of the Cauchy problem

$$\begin{cases} y'(t) = Ay(t) + e^{i\mu} y_0 \\ y(0) = 0, \end{cases}$$

is bounded.

Evidence of the aforementioned theorem can be found, for instance, in [1]. Similar result was been extended for strongly continuous bounded semigroups. We can refer the reader to [39]. For this aim, if the complex Banach space X is endowed with the norm $\|\cdot\|$, and $\mathbf{D} = \{D(t),\ t \geq 0\}$ defines a strongly continuous semigroup acting on the space X having $(G, D(G))$ as the infinitesimal generator (here $D(G)$ denotes the domain of the operator G), Jan van Neerven proved, in [38, Corollary 5], that the semigroup $\mathbf{D}$ is exponentially stable, that is there exist two positive constants $N \geq 1$ and $\nu > 0$