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REMARKS ON THE EXISTENCE AND UNIQUENESS OF PERIODIC SOLUTION FOR Nth-ORDER FUNCTIONAL DIFFERENTIAL EQUATIONS WITH INFINITE DELAY

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Abstract. In this work we apply the coincidence degree theory to establish a result on the existence and uniqueness of periodic solution for a class of nth-order functional differential equations with infinite delay. An example is given to illustrate our result.

Keywords. Periodic solution; existence and uniqueness; coincidence degree; functional differential equations; infinite delay.

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1 Introduction

In this note we consider the nth-order functional differential equation with infinite delay:

$$x^{(n)}(t) = f\left(t, x_t^{(n-1)}, x_t^{(n-2)}, \dots, x'_t, x_t\right),\ t \in \mathbb{R}, \tag{1.1}$$

where

* f is a continuous real function defined in

$$\mathbb{R} \times \underbrace{C_B((-\infty,0],\mathbb{R}) \times \ldots \times C_B((-\infty,0],\mathbb{R})}_{n\ times},$$

  T- periodic in the first argument with $T > 0$, where $C_B((-\infty,0],\mathbb{R})$ denotes the space of the bounded continuous functions $\phi : (-\infty,0] \to \mathbb{R}$ endowed with the norm $\|\phi\|_\infty = \sup_{\tau\in(-\infty,0]} |\phi(\tau)|$;

* x_t represents the mapping $x_t : (-\infty,0] \to \mathbb{R}$ defined by $x_t(\tau) = x(t+\tau)$ for $\tau \in (-\infty,0]$, where $t \in \mathbb{R}$;