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MÖNCH'S FIXED POINT THEOREM ON ADMISSIBLE SPACES

Donal O'Regan

School of Mathematics, Statistics and Applied Mathematics
National University of Ireland, Galway, Ireland

Abstract. In this paper we present fixed point results for compact and noncompact maps defined on admissible or dominating type spaces.

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1 Introduction

In this paper we present a variety of fixed point results for classes of compact and noncompact maps [6, 7, 9, 11] in very general spaces. Our spaces include extension spaces like $ES(\text{compact})$ spaces (so includes absolute retracts), $AES(\text{compact})$ spaces and more generally admissible and dominated spaces. In particular we present a generalization of Mönch's fixed point theorem in a very general setting.

For the remainder of this section we present some definitions and known results which will be needed throughout this paper. Suppose X and Y are topological spaces. Given a class $\mathcal{X}$ of maps, $\mathcal{X}(X,Y)$ denotes the set of maps $F: X \to 2^Y$ (nonempty subsets of Y) belonging to $\mathcal{X}$, and $\mathcal{X}_c$ the set of finite compositions of maps in $\mathcal{X}$. We let

$$\mathcal{F}(\mathcal{X}) = \{Z : \ Fix\,F \neq \emptyset \ \text{ for all } \ F \in \mathcal{X}(Z,Z)\}$$

where $Fix\,F$ denotes the set of fixed points of F.

The class $\mathcal{A}$ of maps is defined by the following properties:

(i). $\mathcal{A}$ contains the class $\mathcal{C}$ of single valued continuous functions;

(ii). each $F \in \mathcal{A}_c$ is upper semicontinuous and closed valued; and

(iii). $B^n \in \mathcal{F}(\mathcal{A}_c)$ for all $n \in \{1, 2, ...\}$; here $B^n = \{x \in \mathbf{R}^n : \ \|x\| \leq 1\}$.

Remark 1.1. The class $\mathcal{A}$ is essentially due to Ben-El-Mechaiekh and Deguire [5]. $\mathcal{A}$ includes the class of maps $\mathcal{U}$ of Park ($\mathcal{U}$ is the class of maps defined by (i), (iii) and (iv). each $F \in \mathcal{U}_c$ is upper semicontinuous and compact valued). Thus if each $F \in \mathcal{A}_c$ is compact valued the class $\mathcal{A}$ and $\mathcal{U}$ coincide.