

## ON AN EXPONENTIAL TYPE SYSTEM OF DIFFERENCE EQUATIONS

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**Abstract.** We study the boundedness and persistence, existence and uniqueness of positive equilibrium point, local and global asymptotic stability of an exponential system of rational difference equations in the interior of  $\mathbb{R}_+^2$ . Some numerical examples are given so as to verify not only the theoretical results but also to have the appearance of stable invariant close curve. The appearance of close invariant curve implies that the discrete-time exponential system undergoes a supercritical Neimark-Sacker bifurcation when parameter varies in a small neighborhood of the unique positive equilibrium point.

**Keywords.** difference equations, boundedness, persistence, local and global stability, numerical simulations

**AMS (MOS) subject classification:** 39A10, 40A05

### 1 Introduction

El-Metwally *et. al.* [1] have investigated the dynamics of following exponential difference equation:

$$x_{n+1} = \alpha + \beta x_{n-1} e^{-x_n}, \quad n = 0, 1, \dots, \quad (1)$$

where  $\alpha$ ,  $\beta$  and  $x_0$ ,  $x_{-1}$  are arbitrary non-negative real numbers. Equation (1) may be viewed as a model in Mathematical Biology, where  $\alpha$  be the immigration rate and  $\beta$  be the population growth rate. Ozturk *et. al.* [2] have investigated the dynamics of following exponential difference equation:

$$x_{n+1} = \frac{\alpha + \beta e^{-x_n}}{\gamma + x_{n-1}}, \quad n = 0, 1, \dots, \quad (2)$$

where  $\alpha$ ,  $\beta$ ,  $\gamma$  and  $x_0$ ,  $x_{-1}$  are arbitrary non-negative numbers. Bozkurt [3] has investigated the local and global behavior of following exponential difference equation:

$$x_{n+1} = \frac{\alpha e^{-x_n} + \beta e^{-x_{n-1}}}{\gamma + \alpha x_n + \beta x_{n-1}}, \quad n = 0, 1, \dots, \quad (3)$$