

TIME-OPTIMAL CONTROL PROBLEM FOR TIME-FRACTIONAL DIFFERENTIAL SYSTEM USING DUBOVITSKII-MILYUTIN METHOD

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Abstract. A time-optimal control problem for time-Fractional Differential System (FDS) is considered. The fractional time derivative is considered in the Caputo sense. A time optimal control problem is replaced by an equivalent one with a performance index in the integral form. We first study the existence and the uniqueness of the solution of the fractional differential system in a Hilbert space. Then we show that the considered optimal control problem has a unique solution. Constraints on controls are assumed. To obtain the boundary optimality conditions for the Neumann problem, the generalization of the Dubovitskii-Milyutin Theorem was applied. Some illustrate examples are analyzed in details.

Keywords. Time optimal control problems, Fractional optimal control, Neumann parabolic equations, Dubovitskii-Milyutin theorem, Conical approximations, Boundary optimality conditions, Weierstrass theorem.

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1 Introduction

Fractional calculus has received considerable attention in recent years, largely because it has been demonstrated that in many fields of science, engineering, economics, bioengineering and applied mathematics, fractional derivatives can be used to develop accurate models of many phenomena of nature. Several books have been written recently in this area, of which we recommend ([30], [35], [34]) to researchers in science and engineering entering to this field for the first time. One of the areas where considerable progress has been made is the area of fractional variational calculus. Riewe [36], [37] was the first to initiate this field. Subsequently, Agrawal [1] brought it to the main stream and initiated the fractional calculus of variations field. Since then, a large number of researchers have followed the lead of these work, and applied it to various fields (see [15]- [23] and references in those papers).