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THE GEOMETRY OF THE REGION OF UNIFORM EXPONENTIAL STABILITY FOR LINEAR TIME INVARIANT DYNAMIC EQUATIONS ON TIME SCALES

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Abstract. Doan *et al.* [6, 5] recently examined the region of uniform exponential stability for first order, linear time invariant dynamic equations on time scales. In particular, they examined the largest disc tangent to the origin contained the region of uniform exponential stability, $\mathcal{US}$, as defined by Pötzsche *et al.* [9]. Both of these sets, while theoretically important, are not well understood outside of a narrow class of time scales. Here, we provide a complete description of the largest disc tangent to the origin contained the region of exponential stability. Furthermore, we provide a condition on the time scale – *mean-stationarity* – which guarantees that this disc is a subset of the region of uniform exponential stability.

Keywords. Time Scales; Exponential Stability; Uniform Exponential Stability; Pole-Placement; Hybrid Systems

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1 Introduction

A fundamental question in time scales theory is the properties of the *time scales exponential* function which is defined as the solution to the time scales initial value problem

$$x^{\Delta}(t) = \lambda x(t); \qquad x(t_0) = x_0; \qquad \lambda \in \mathbb{C} \qquad t \in \mathbb{T}. \tag{1}$$

In particular, conditions for the stability (Lyapunov, exponential, asymptotic, global, uniform, robust) of (1) is of interest to engineers for the design of observers and controllers [2, 8, 10]. Pötzsche *et al.* [9] showed that placing λ in a certain region of the complex plane was necessary and sufficient for exponential stability of (1). Additionally, they showed how to calculate the