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TRAJECTORY BASED MARKET MODELS
Evaluation of Minmax Price Bounds

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Abstract. The paper studies sub and super-replication price bounds for contingent claims defined on general trajectory based market models. No prior probabilistic or topological assumptions are placed on the trajectory space which is of unrestricted cardinality. For a given option, there exists an interval bounding the set of possible fair prices; such interval exists under more general conditions than the usual no-arbitrage requirement. The paper develops a backward recursive method to evaluate the option bounds together with the associated hedging strategies; the global minmax optimization, defining the price interval, is reduced to a local minmax optimization via dynamic programming. Trajectory sets are introduced for which existing probabilistic and non-probabilistic market models are nested as particular cases. Several examples are presented, the effect of the presence of arbitrage on the price bounds is illustrated.

Keywords. non-probabilistic market models, arbitrage, fair price bounds, minmax optimization, dynamic programming, hedging.

AMS (MOS) subject classification: 91G10, 91G20, 49J35, 49K35, 49L20.

1 Introduction

In an incomplete market model, the classical (stochastic) theory shows that, under no arbitrage assumptions, there exists an interval of fair prices. Such an interval is given by the sub and super-replication bounds introduced first in a diffusion setting in [12] (see [15] for a general discrete time formulation). The super-replication price bound of an European contingent claim Z equals the supremum of its expectation over the set of equivalent martingale measures (with an analogous characterization for sub-replication.) For a discrete time