

ON SOLUTIONS OF A SYSTEM OF TWO FOURTH-ORDER DIFFERENCE EQUATIONS

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Abstract. In this paper, we investigate the dynamical behavior of the positive solutions of the system of difference equations

$$u_{n+1} = \frac{au_{n-1}}{b + cv_{n-3}^p}, \quad v_{n+1} = \frac{dv_{n-1}}{e + fu_{n-3}^q}, \quad n \in \mathbb{N}_0,$$

where the initial conditions u_{-i}, v_{-i} ($i = 0, 1, 2, 3$) are non-negative real numbers and the parameters a, b, c, d, e, f, p, q are positive real numbers, by extending some results in the literature.

Keywords. System of difference equations; stability; global behavior; periodic solution.

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1 Introduction

Recently, difference equations and their systems have gained a great importance. Most of the recent applications of these equations have appeared in many scientific areas such as biology, ecology, physics, engineering, economics. Particularly, rational difference equations and their systems have great importance in applications. So, it is very worthy to examine the behavior of solutions of a system of rational difference equations and to discuss the stability character of their equilibrium points. In recent years, many researchers have investigated global behavior of solutions of non-linear difference equations and their two or three dimensional systems. For example, El-Owaidy et al. [10] investigated global behavior of positive solutions of the difference equation

$$x_{n+1} = \frac{\alpha x_{n-1}}{\beta + \gamma x_{n-2}^p}, \quad n \in \mathbb{N}_0, \quad (1)$$

with non-negative parameters and non-negative initial conditions. Gumus and Soykan [11] studied the dynamical behavior of the positive solutions for a system of rational difference equations of the following form

$$u_{n+1} = \frac{\alpha u_{n-1}}{\beta + \gamma v_{n-2}^p}, \quad v_{n+1} = \frac{\alpha_1 v_{n-1}}{\beta_1 + \gamma_1 u_{n-2}^p}, \quad n \in \mathbb{N}_0, \quad (2)$$