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FORBIDDEN SET AND SOLUTIONS OF A HIGHER ORDER DIFFERENCE EQUATION

R. Abo-Zeid

Department of Basic Science
The Higher Institute for Engineering & Technology, Al-Obour, Cairo, Egypt
email: abuzead73@yahoo.com

Abstract. In this paper, we determine the forbidden set, introduce an explicit formula for the solutions and discuss the global behavior of solutions of the difference equation

$$x_{n+1} = \frac{ax_n x_{n-k}}{bx_n + cx_{n-k-1}}, \quad n = 0, 1, \ldots$$

where a, b, c are positive real numbers, the initial conditions $x_{-k-1}, x_{-k}, ..., x_{-1}, x_0$ are real numbers and k is a nonnegative integer.

Keywords. difference equation, explicit formula, forbidden set, periodic solution, unbounded solution.

AMS (MOS) subject classification: 39A10

1. Introduction

Difference equations have played an important role in analysis of mathematical models of biology, physics and engineering. Recently, there has been a great interest in studying properties of nonlinear and rational difference equations. One can see [6, 9, 12, 14, 15, 16, 17, 18, 20, 22] and the references cited therein.

The study of nonlinear difference equations that having quadratic terms is not easy and worth to be discussed. Results concerning rational difference equations having quadratic terms are included in [3, 4, 5, 7, 8, 10, 11, 13, 19, 21, 23, 24, 25] and the references cited therein.

In [19], H. Sedaghat determined the global behavior of all solutions of the rational difference equations

$$x_{n+1} = \frac{ax_{n-1}}{x_n x_{n-1} + b}, \quad x_{n+1} = \frac{ax_n x_{n-1}}{x_n + bx_{n-2}}, \quad n = 0, 1, \ldots$$

where $a, b > 0$.

In [1], we investigated the global behavior of all solutions of the rational difference equation

$$x_{n+1} = \frac{ax_n x_{n-1}}{-bx_n + cx_{n-2}}, \quad n = 0, 1, \ldots$$

where a, b, c are positive real numbers and the initial conditions x_{-2}, x_{-1}, x_0 are real numbers.