

A STUDY OF NONLOCAL INTEGRO-MULTI-POINT BOUNDARY VALUE PROBLEMS OF SEQUENTIAL FRACTIONAL INTEGRO-DIFFERENTIAL INCLUSIONS

Bashir Ahmad¹, Sotiris K. Ntouyas^{1,2}, Ahmed Alsaedi¹
and Ravi P. Agarwal³

¹Department of Mathematics, NAAM-Research Group, Faculty of Science, King
Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia

² Department of Mathematics, University of Ioannina, 451 10 Ioannina, Greece

³Department of Mathematics, Texas A&M University, Kingsville, TX 78363-8202, USA

Abstract. We develop the existence criteria for the solutions of Caputo type sequential fractional integro-differential inclusions equipped with nonlocal multi-point boundary conditions involving Riemann-Liouville integral of fractional-order. We take into account the convex and non-convex multivalued maps in our analysis and make use of fixed point theorems for multivalued maps to establish the desired results.

Keywords. Sequential fractional derivative; inclusions; multi-point; integral boundary conditions; fixed point.

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1 Introduction

In this paper, we study the existence of solutions for a nonlocal integro-multi-point boundary value problem of sequential fractional integro-differential inclusions given by

$$({}^c D^q + k {}^c D^{q-1})x(t) \in F(t, x(t), {}^c D^\beta x(t), I^\gamma x(t)), \quad t \in [0, 1], \quad 2 < q \leq 3, \quad (1)$$

$$x(0) = 0, \quad x'(0) = 0, \quad \sum_{i=1}^m a_i x(\zeta_i) = \lambda \int_0^\eta \frac{(\eta-s)^{\delta-1}}{\Gamma(\delta)} x(s) ds, \quad \delta > 0, \quad (2)$$

where ${}^c D^{(\cdot)}$ denotes the Caputo fractional derivative of order (\cdot) , $I^{(\cdot)}$ denotes the Riemann-Liouville integral of fractional order (\cdot) , $F: [0, 1] \times R^3 \rightarrow \mathcal{P}(R)$ is a multivalued map, $\mathcal{P}(R)$ is the family of all nonempty subsets of R , $0 < \beta, \gamma < 1, k > 0, 0 < \eta < \zeta_1 < \dots < \zeta_m < 1$ and $\lambda, a_i, i = 1, \dots, m$ are real constants.