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INVARIANCE ANALYSIS OF A THIRD-ORDER DIFFERENCE EQUATION WITH VARIABLE COEFFICIENTS

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Abstract. Non-trivial symmetries of the third-order difference equations of the form

$$u_{n+3} = \frac{u_n u_{n+1}}{u_{n+2}(A_n + B_n u_n u_{n+1})}$$

are obtained. These symmetries are used to investigate their solutions for some random sequences (A_n) and (B_n). We extend the results obtained in [7] and conditions for well-defined solutions of the equations under investigation are obtained.

Keywords. Difference equation, symmetry, reduction, group invariant solutions, periodicity

AMS (MOS) subject classification: 39A10, 39A99, 39A13

1 Introduction

Over a century ago, there were a number of techniques used to solve ordinary differential equations. Marius Sophus Lie (1842-1899) unified and extended these techniques after realizing that they were special cases of a general integration procedure based on the invariance of ordinary differential equations under a continuous group of symmetries [10]. In 1987, Shigeru Maeda showed that the Lie's method extension can also be used to solve ordinary difference equations. He also showed that the set of functional equations amounted from the linearized symmetry condition of the ordinary difference equation [11]. Later, several authors have studied ordinary difference equations and some interesting results have been obtained - see [1–6, 8, 9, 12, 13] and references therein. In [7], T.F. Ibrahim and N. Touafek studied the equation

$$x_{n+1} = \frac{x_{n-1}x_{n-2}}{x_n\,(a_n + b_n x_{n-1}x_{n-2})}, \tag{1}$$

where $(a_n)_{n\in\mathbb{N}_0}$, $(b_n)_{n\in\mathbb{N}_0}$ are real two-periodic sequences and the initial values x_{-2}, x_{-1}, x_0 are non-zero real numbers.

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