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ON NEUTRAL FIRST ORDER DELAY DIFFERENTIAL EQUATIONS WITH M COMMENSURATE DELAYS

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Abstract. In this paper we derive a robust algorithmic stability test to determine asymptotic stability of the zero solution of a first order neutral differential equation of the form

$$y'(t) + \alpha y'(t-\ell\tau) + \sum_{j=0}^{m} a_j y(t-j\tau) = 0$$

where $a_j,\ j=0,\ldots,m$, α and τ are constants, and $|\alpha|<1$, $\tau>0$, ℓ is a positive integer less than or equal to m. New necessary conditions for asymptotic stability are also obtained when $\ell=1$ or $\ell=2$. In addition, we obtain an algorithmic stability test. In proving our results, we make use of Pontryagin's theory for quasi-polynomials and Chebyshev polynomials.

Keywords. stability criteria, algorithmic stability test, commensurate delays, characteristic functions, Chebyshev polynomials.

1 Introduction

The aim of this paper is to derive a robust algorithmic stability test to determine asymptotic stability of the zero solution of the neutral differential equation of the form

$$y'(t) + \alpha y'(t-\ell\tau) + \sum_{j=0}^{m} a_j y(t-j\tau) = 0 \tag{1}$$

where $a_j,\ j=0,\ldots,m$, α and τ are constants, $|\alpha|<1$, $\tau>0$. The case $\ell=m$ was considered in our previous paper [3]. In [3], we obtained a very convenient necessary condition using Chebyshev polynomials for asymptotic stability when $\ell=m$ but only for $m\le 9$. The case for $m>9$ when $\ell=m$