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ROBUSTLY TRANSITIVE SETS WITH SHADOWING FOR VECTOR FIELDS

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Abstract. Let M be a closed $n(\geq 3)$-dimensional smooth Riemannian manifold, and let X be a C^1-vector field on M. We prove that for a robustly transitive X if X has the C^1-robustly shadowing property then $Sing(X) = \emptyset$ and it is Anosov. Moreover, C^1-generically, a robustly transitive X if X has the shadowing property then $Sing(X) = \emptyset$ and it is Anosov.

Keywords. shadowing, robustly transitive, homogeneous, generic, hyperbolic, Anosov.

AMS (MOS) subject classification: Primary 37C50, 37C10; Secondary 34D10, 37C20.

1 Introduction

Many of the dynamic results obtained from diffeomorphisms can be extended to the case of vector fields, but not always, in particular results involve the hyperbolic structure. For instance, it is well known that if a diffeomorphism $f : M \to M$ has a C^1-neighborhood $\mathcal{U}(f)$ such that every periodic point of $g \in \mathcal{U}(f)$ is hyperbolic, then the nonwandering set $\Omega(f)$ is hyperbolic (see [7]). However, the result is not true for the case of vector fields (see [6]). We consider a robustly transitive set. That is a main topic in the dynamical systems. In fact, Mañé [13] showed that every robustly transitive diffeomorphism on M, in a two-dimension manifold is an Anosov diffeomorphism. Díaz *et al* [4] proved that if in three-dimensional manifold M, robustly transitive diffeomorphisms are partially hyperbolic and Bonatti *et al* [3] showed that every robustly transitive diffeomorphism M in any dimensional manifold admits a dominated splitting. Recently, Lee [9] proved that if a robustly chain transitive diffeomorphism which has same indices then it is hyperbolic. From these results, Lee [10] proved that if a robustly chain transitive diffeomorphism has the orbital shadowing property then it is hyperbolic. Lee and Park [12] proved that if a robustly chain transitive diffeomorphism has the shadowing property then it is hyperbolic homoclinic class. For vector fields, Doering [5] showed that if a compact manifold M is three-dimension then every robustly