

THE STABILITY OF SYSTEMS OF DIFFERENCE EQUATION WITH NON-CONSISTENT INITIAL CONDITIONS

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Abstract: For given non-consistent initial conditions, we study the stability of a class of generalised linear systems of difference equations with constant coefficients and taking into account that the leading coefficient can be a singular matrix. We focus on the optimal solutions of the system and derive easily testable conditions for stability.

Keywords: singular system, stability, difference equations, optimal, non-consistent.

1 Introduction

Singular systems of difference/differential equation have been studied by many authors in the past years. See [1-30] for qualitative analysis of this type of systems such as Theorems for existence of solutions, stability results, and [31-40] for recent applications of such systems. For an extended version of this type of systems using fractional operators, see [41 – 50]. In this article, we consider the following initial value problem:

$$\begin{aligned} FY_{k+1} &= GY_k, \quad k = 1, 2, \dots, \\ Y_0. \end{aligned} \tag{1}$$

Where $F, G \in \mathbb{R}^{m \times m}$ and $Y_k \in \mathbb{R}^m$. The matrix F is singular ($\det F=0$). The initial conditions Y_0 are considered to be non-consistent. Note that the initial conditions are called consistent if there exists a solution for the system which satisfies the given conditions. We will also assume that the pencil of the system is regular, i.e. for an arbitrary $s \in \mathbb{C}$ we have $\det(sF - G) \neq 0$, see [51 – 58].

There are stability results in the literature dealing with regular systems and for generalised systems with consistent conditions, see [12, 13, 28, 47]. As already mentioned, we consider initial conditions that are non-consistent.