THE STABILITY OF SYSTEMS OF DIFFERENCE EQUATION WITH NON-CONSISTENT INITIAL CONDITIONS

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Abstract: For given non-consistent initial conditions, we study the stability of a class of generalised linear systems of difference equations with constant coefficients and taking into account that the leading coefficient can be a singular matrix. We focus on the optimal solutions of the system and derive easily testable conditions for stability.

Keywords: singular system, stability, difference equations, optimal, non-consistent.

1 Introduction

Singular systems of difference/differential equation have been studied by many authors in the past years. See [1-30] for qualitative analysis of this type of systems such as Theorems for existence of solutions, stability results, and [31-40] for recent applications of such systems. For an extended version of this type of systems using fractional operators, see [41 – 50]. In this article, we consider the following initial value problem:

\begin{equation}
FY_{k+1} = GY_k, \quad k = 1, 2, \ldots, Y_0.
\end{equation}

Where \(F, G \in \mathbb{R}^{m \times m}\) and \(Y_k \in \mathbb{R}^m\). The matrix \(F\) is singular (\(\det F = 0\)). The initial conditions \(Y_0\) are considered to be non-consistent. Note that the initial conditions are called consistent if there exists a solution for the system which satisfies the given conditions. We will also assume that the pencil of the system is regular, i.e. for an arbitrary \(s \in \mathbb{C}\) we have \(\det (sF - G) \neq 0\), see [51 – 58].

There are stability results in the literature dealing with regular systems and for generalised systems with consistent conditions, see [12, 13, 28, 47]. As already mentioned, we consider initial conditions that are non-consistent.