

## STOCHASTIC STABILITY OF FRACTIONAL MAXWELL VISCOELASTIC SYSTEMS

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**Abstract.** The stochastic stability of a single degree-of-freedom fractional viscoelastic system under bounded noise excitation is investigated through moment Lyapunov exponents by using the method of higher-order stochastic averaging. A realistic example of such system is the transverse vibration of a viscoelastic column under the excitation of stochastic axial compressive load. The viscoelastic material is assumed to follow a fractional Maxwell constitutive relation. The method of higher-order stochastic averaging is developed to approximate the fractional stochastic differential equation of motion, and then moment Lyapunov exponents are determined for the system with small damping and weak random fluctuation. Results show that for this viscoelastic structure, second-order averaging analysis is adequate for stability analysis. The effects of various parameters on the stochastic stability are discussed.

**Keywords.** Stochastic stability, moment Lyapunov exponents, bounded noise, fractional Maxwell viscoelasticity, stochastic averaging.

### 1 Introduction

Deterministic dynamic stability of viscoelastic systems has been intensively investigated. However, loadings from earthquakes, explosion, wind, and ocean waves can be described satisfactorily only in probabilistic domain, which results in the equation of motion of the viscoelastic system under such excitation usually governed by stochastic integro-differential equation [1]. The response and stability of the system are difficult to be obtained exactly. Therefore, several numerical and approximate procedures have been proposed. The method of stochastic averaging has been widely used to approximately solve stochastic differential equations containing a small parameter. The basic idea of stochastic averaging method is to approximate the original Stratonovich stochastic system by an averaged Itô stochastic system (diffusive Markov process), which is presumably easier to study, and infer properties of the dynamics of the original system by the understanding of the dynamics of the averaged system. It has been shown that the ordinary stochastic averaging is a first order approximation method. Ariaratnam [2] applied the method of stochastic averaging to study stochastic stability of linear viscoelastic systems. Deng et al. [3] investigated stochastic stability of a fractional viscoelastic column under bounded noise excitation using first-order stochastic averaging method. Huang and Xie extended