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EVOLUTION SEMIGROUPS AND SPECTRAL CRITERIA FOR ASYMPTOTICALLY ALMOST PERIODIC SOLUTIONS OF DISCRETE PERIODIC EVOLUTION EQUATIONS

Akbar Zada and Muhammad Arif

Department of Mathematics
University of Peshawar, Peshawar, Pakistan

Abstract. Let q be a positive integer greater or equal to 2 and let $\mathcal{X}$ be a complex Banach space. We denote by $\mathcal{Z}_+$ the set of all nonnegative integers. Let $\mathcal{C}_{00}(\mathcal{Z}_+, \mathcal{X})$ is the space of all $\mathcal{X}$-valued bounded sequences which decays at zero and at infinity and $\mathcal{AP}_0(\mathcal{Z}_+, \mathcal{X})$ is the space of all $\mathcal{X}$-valued almost periodic sequences decaying at zero. Then we consider the space $\mathcal{AAP}_0(\mathcal{Z}_+, \mathcal{X})$ as the direct sum of $\mathcal{C}_{00}(\mathcal{Z}_+, \mathcal{X})$ and $\mathcal{AP}_0(\mathcal{Z}_+, \mathcal{X})$. We prove that the discrete evolution family $\mathcal{U} = \{\mathbb{U}(m, n) : m, n \in \mathcal{Z}_+,\ m \geq n\}$ is uniformly exponentially stable if and only if for each $z(n) \in \mathcal{AAP}_0(\mathcal{Z}_+, \mathcal{X})$ the solution of the Cauchy problem

$$X = \begin{cases} y_{n+1} = \mathcal{A}_n y_n + z(n+1), \\ y(0) = 0, \end{cases} \tag{0.1}$$

belongs to $\mathcal{AAP}_0(\mathcal{Z}_+, \mathcal{X})$.
Our proof uses the approach of discrete evolution semigroups.

Keywords. Discrete evolution semigroup, Discrete evolution family, Uniform exponential stability, Almost periodic sequences, Asymptotically almost periodic sequences.

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1 Introduction

Let us consider the following linear discrete evolution equations

$$\zeta_{n+1} = \mathcal{A}_n \zeta_n$$

and

$$\zeta_{n+1} = \mathcal{A}_n \zeta_n + f_n,$$

where $\zeta_n \in \mathcal{X}$, $\mathcal{X}$ is a complex Banach space. The study of the difference equations $\zeta_{n+1} = \mathcal{A}_n \zeta_n$ or $\zeta_{n+1} = \mathcal{A}_n \zeta_n + f_n$ leads to the idea of discrete evolution family. For such systems the asymptotic behavior of solutions at infinity is of particular interest, the corresponding continuous systems has been a central topic discussed for such behavior from the last few decades. We refer the reader to [1, 4, 5, 8–10, 13, 14, 18] and references therein for more complete information on the subject.