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A PIECEWISE QUADRATIC MAXIMUM ENTROPY METHOD FOR INVARIANT MEASURES OF POSITION DEPENDENT RANDOM MAPS

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Let $\{\tau_1, \tau_2, \cdots, \tau_K\}$ be a collection of nonsingular maps on $[0,1]$ into $[0,1]$ and $\{p_1, p_2, \cdots, p_K\}$ be a collection of position dependent probabilities on $[0,1]$. We consider position dependent random maps $T = \{\tau_1, \tau_2, \cdots, \tau_K; p_1, p_2, \cdots, p_K\}$ such that T preserves an absolutely continuous invariant measures with density f^*. A piecewise quadratic maximum entropy method for approximating f^* is developed. We present a proof of convergence of the maximum entropy method for random maps. We also present a numerical example.

Keywords. Boltzmann Entropy; Piecewise Quadratic Maximum Entropy Method; Position Dependent Random Maps; Frobenius-Perron Operator; Absolutely Continuous Invariant Measures;

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1 Introduction

In this paper, we present Boltzmann entropy as a mathematical tool for the approximation of invariant measures for position dependent random maps. The existence and properties of invariant measures of dynamical systems reflect their long time behavior and play an important role in understanding their statistical properties and chaotic nature. It is well known that fixed points of Frobenius-Perron operator of a dynamical system are invariant densities of absolutely continuous invariant measures of the system. The Frobenius-Perron equation is a functional equation and it is difficult to solve this equation except in some simple cases. Approximation of invarianr measures is one of the major research topics in ergodic theory and dynamical systems.

Rudolf Clausius introduced the concept of entropy into thermodynamics in 1865. In 1866, Ludwig Boltzmann introduced a different form of entropy in his pioneering work on the kinetic theory of gases published in [17]. Since then, entropy has played an important role in the development of many