

BIRKHOFF–KELLOGG AND FURI–PERA TYPE RESULTS FOR GENERAL CLASSES OF MAPS

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Abstract. This paper presents a variety of Birkhoff–Kellogg type theorems and a Furi–Pera type result for a general class of maps.

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1 Introduction

Epi maps were introduced in [4] and extended by a variety of authors (see for example [6, 8]). Using the notion of a Φ -epi map (see [8]) we establish a variety of Birkhoff–Kellogg type results for general classes of maps. In particular our results include those in [2, 3, 7, 10]. In addition in this paper we present a Furi–Pera type result for general classes of maps which improve results in the literature (see [1, 5, 9]).

2 Main results

In this section we present Birkhoff–Kellogg type results and a Furi–Pera type result.

Let E be a normal topological vector space and U an open subset of E . We first consider the classes **A** and **B** of maps.

Definition 2.1. We say $F \in A(\bar{U}, E)$ (respectively $F \in MB(\bar{U}, E)$) if $F : \bar{U} \rightarrow 2^E$ and $F \in \mathbf{A}(\bar{U}, E)$ (respectively $F \in \mathbf{B}(\bar{U}, E)$); here \bar{U} denotes the closure of U in E and 2^E denotes the family of nonempty subsets of E .

Remark 2.1. We say $F \in MB(E, E)$ if $F : E \rightarrow 2^E$ and $F \in \mathbf{B}(E, E)$.

In this section we fix a $\Phi \in MB(\bar{U}, E)$.

Definition 2.2. We say $F \in A_{\partial U}(\bar{U}, E)$ if $F \in A(\bar{U}, E)$ with $F(x) \cap \Phi(x) = \emptyset$ for $x \in \partial U$; here ∂U denotes the boundary of U in E .