

BEHAVIOR OF TWO AND THREE-DIMENSIONAL SYSTEMS OF DIFFERENCE EQUATIONS IN MODELLING COMPETITIVE POPULATIONS

T. F. Ibrahim

Mathematics Department, Faculty of Science , Mansoura University , Egypt

Abstract. The main objective of this paper is to determine the closed form solutions of system of non-linear difference equations in the modelling competitive populations

$$x_{n+1} = \frac{x_{n-1}}{a_n + x_{n-1} y_n}, y_{n+1} = \frac{y_{n-1}}{a_n + y_{n-1} x_n}, z_{n+1} = \frac{1}{y_n z_n}$$

where $\{a_n\}$ sequence with initial values $x_{-1}, x_0, y_{-1}, y_0,$ and z_0 such that $x_{-1} y_0 \neq -a_0, x_0 y_1 \neq -a_1, y_{-1} x_0 \neq -a_0, ,$ and $y_0 x_1 \neq -a_1$. We study some special cases of system (1). In the Final section ,we introduce numerical examples.

Keywords. solutions, difference equation, competitive, convergence, stability

AMS (MOS) subject classification: 39A10, 39A11

1 Introduction

Discrete dynamical systems or Difference equations [35] is varied field which manipulate nearly every offshoot of applied and pure mathematics. The studying of properties of systems of difference equations is a subject of interest in last years, see books [[1], [11], [17]]. A difference equations system of order one

$$S_{n+1} = \phi(S_n, T_n) \quad , \quad T_{n+1} = \psi(S_n, T_n) \quad (1)$$

, $n = 0, 1, \dots, (S_0, T_0) \in R$, R subset of $\mathbb{R}^2, (\phi, \psi) : R \rightarrow R$, and ϕ, ψ are mapping is *competitive* if $\phi(s, t)$ is non-decreasing in s and non-increasing in t ; and $\psi(s, t)$ is nondecreasing in t and nonincreasing in s [36].

Competitive systems deliberated by plentiful authors (For examples [3], [4], [8], [15], [18], [26], [36] , [37], [38]).

In a modelling framework, competitive system of nonn-linear difference equations

$$s_{n+1} = \frac{s_n}{a + t_n} \quad \& \quad t_{n+1} = \frac{t_n}{b + s_n}$$