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ON TWO NONLINEAR DIFFERENCE EQUATIONS

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Abstract. The behavior of solutions of the following nonlinear difference equations

$$x_{n+1} = \frac{q}{p + x_n^{\nu}} \quad \text{and} \quad y_{n+1} = \frac{q}{-p + y_n^{\nu}},$$

with real nonzero initial conditions x_0 and y_0, where $p, q \in \mathbb{R}^+$ and $\nu \in \mathbb{N}$, is studied. The solution forms of these two equations when $\nu = 1$ are expressed in terms of Horadam numbers. Meanwhile, the behavior of their solutions is investigated for all integers $\nu > 1$ and several numerical examples are presented to illustrate the results exhibited. The present work generalizes those seen in [*Adv. Differ. Equ.*, **2013**:174 (2013), 7 pages].

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1 Introduction

An equation of the form

$$x_{n+1} = f(x_n, x_{n-1}, \dots, x_{n-k}), \quad n = 0, 1, \dots \tag{1}$$

where f is a continuous function that maps some set I^{k+1} into I is called a *difference equation of order* $k+1$. The set I is usually a sub-interval of the set of real numbers $\mathbb{R}$, a union of its sub-intervals, or a discrete subset of $\mathbb{R}$ such as the set of *integers* $\mathbb{Z}$. A solution of (1), uniquely determined by a prescribed set of $(k+1)$ *initial conditions* $x_{-k}, x_{-k+1}, \dots, x_0 \in I$, is a sequence $\{x_n\}_{n=-k}^{\infty}$ that satisfies equation (1) for all $n \geq 0$. If for some least value $m \geq -k$, an initial point $(x_{-k}, x_{-k+1}, \dots, x_0) \in I^{k+1}$ generates a solution $\{x_n\}$ with undefined value x_m, then we call the set S of all such points the *singularity set*, which also called the ‘‘forbidden set’’ in the literature [3, 9]. On the other hand, a solution of equation (1), which is constant for all $n \geq -k$, is called an *equilibium solution* of (1). If $x_n = \bar{x}$ for all $n \geq -k$ is an equilibrium solution of (1), then $\bar{x}$ is called an *equilibrium point*, or simply