Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **24 (2017)** 347-373

A VARIETY OF INVERSE HILBERT TYPE INEQUALITIES ON TIME SCALES

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Abstract. Using algebraic inequalities, the reverse Hölder inequality, the reverse Jensen inequality and a simple consequence of Keller's chain rule on time scales, we prove some new inverse dynamic inequalities of Hilbert type on a time scales $\mathbb{T}$.

Keywords. Hilbert's inequality**,** Hölder's inequality, Jensen's inequality, time scales.

AMS (MOS) subject classification: 6D15, 34A40, 39A12, 34N05.

1 Introduction

The classical Hilbert inequality states that for $f(x)$, $g(x) \geq 0$ with

$$\int_0^\infty f^2(x)dx < \infty, \quad \text{and} \quad \int_0^\infty g^2(x)dx < \infty,$$

then

$$\int_0^\infty \int_0^\infty \frac{f(x)g(y)}{x+y} dxdy \leq \pi \left(\int_0^\infty f^2(x)dx\right)^{1/2} \left(\int_0^\infty g^2(x)dx\right)^{1/2}. \quad (1.1)$$

The constant π is the best possible. The discrete version of inequality (1.1) is given by

$$\sum_{m=1}^\infty \sum_{n=1}^\infty \frac{a_m b_n}{m+n} \leq \pi \left(\sum_{n=1}^\infty a_n^2\right)^{1/2} \left(\sum_{m=1}^\infty b_m^2\right)^{1/2}, \quad (1.2)$$

where $\{a_m\}_{m=1}^\infty$ and $\{b_n\}_{n=1}^\infty$ are nonnegative sequences and π is again the best possible. Hilbert's inequality (1.1) was generalized by Hardy and Riesz