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# SUPPLEMENT TO THE PAPER OF HALIM, TOUAFEK AND ELSAYED: PART II

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**Abstract.** This note is the second of the two supplements to the paper [Closed form solutions of some systems of rational difference equations in terms of Fibonacci numbers, *Dyn. Contin. Discrete Impuls. Syst. Ser. A Math. Anal.*, **21**(6), (2014), 473–486.]. As in the first supplemental note, an alternative proof – more informative and detailed – is presented in this study to explain theoretically the results exhibited in the aforementioned work. The ideas delivered here are of importance because they provide new insights on how to deal with nonlinear difference equations whose solutions are expressible in terms of Fibonacci numbers.

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# 1 Introduction

Our aim in this work is to establish, through an analytical approach, the solution form of the system of nonlinear difference equations

$$x_{n+1} = \frac{(y_{n-3} - x_{n-2})y_n}{y_{n-3} - x_{n-2} + y_n}, \quad y_{n+1} = \frac{(y_{n-2} - x_{n-1})x_{n-1}}{y_{n-2}}, \quad n \in \mathbb{N}_0, \quad (1)$$

which has been examined earlier in [9]. The strategy we used in deriving the solution form provides a clear explanation on how will the Fibonacci numbers

$$(F_n)_0^\infty := (F_n)_{n=0}^\infty = \{0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233 \ldots\}$$

will arise naturally in the solution form of the given system. We have intended to write a separate note for the solution form of system (1) to highlight the technique used in deriving the closed-form solution of the given system.