

CONVERGENCE OF THE SPLITTING METHOD FOR INVERSE PROBLEMS IN PARABOLIC DIFFERENTIAL EQUATIONS

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Abstract. In this article, a splitting method is developed for an inverse problem of determining the unknown parameters and coefficients in parabolic differential equations. The specified data are used and by employing the Lie splitting method, the problem is decomposed into linear and nonlinear parts where each subproblem is solved by numerical algorithms. The convergence of the presented approach is also discussed and proved. Moreover, the efficiency and applicability of the method are demonstrated on various numerical examples which also confirm the theoretical results.

Keywords. Inverse Problem, Operator Splitting Method, Convergence.

AMS (MOS) subject classification: 65M32

1 Introduction

Inverse problems and parameter identification are one of the active fields in science and engineering (see, e.g., [39, 6, 38, 41, 21]) which occur in many physical phenomena, such as, the study of heat conduction processes, thermoelasticity, chemical diffusion, vibration problems and control theory (see, e.g., [3, 2, 7, 10, 9, 4, 12]). The unknown parameters can depend on spatial or time variables (see, e.g., [1, 5, 30, 34]). For ordinary differential equations, the most widely studied inverse problems were formulated as classical Sturm–Liouville problems (see, e.g., [13, 20]), where the unknown coefficients need to be determined from spectral data. The class of inverse problems considered here is based only on boundary measurements (see, e.g., [23, 25, 24]). In this paper, an inverse problem of determining $\{u(t, x), d(t), f(t)\}$ is considered in the one-dimensional parabolic equation

$$\begin{aligned} u_t - a(t, x)u_{xx} + b(t, x)u_x + d(t)u &= f(t)g(t, x) & (t, x) \in Q \\ u(0, x) &= u_0(x), & x \in \Omega = [-l, l], \\ u(t, -l) &= u(t, l) = 0, & t \in [0, T], \end{aligned} \tag{1}$$