

OSCILLATION TESTS FOR DIFFERENCE EQUATIONS WITH NON-MONOTONE ARGUMENTS

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Abstract. Sufficient oscillation conditions for difference equations with non-monotone deviating arguments and nonnegative coefficients are established. These conditions improve all previous well-known results in the literature. Also, using algorithms on MATLAB software, examples illustrating the significance of the results are given.

Keywords. difference equations, non-monotone arguments, retarded arguments, advanced arguments, oscillation, Grönwall inequality.

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1 Introduction

The paper deals with the difference equation with a single variable retarded argument of the form

$$\Delta x(n) + p(n)x(\tau(n)) = 0, \quad n \in \mathbb{N}_0 \quad (\text{E})$$

and the (dual) difference equation with a single variable advanced argument of the form

$$\nabla x(n) - q(n)x(\sigma(n)) = 0, \quad n \in \mathbb{N}, \quad (\text{E}')$$

where \mathbb{N}_0 and \mathbb{N} are the sets of nonnegative integers and positive integers, respectively.

Equations (E) and (E') are studied under the following assumptions: everywhere $(p(n))_{n \geq 0}$ and $(q(n))_{n \geq 1}$ are sequences of nonnegative real numbers, $(\tau(n))_{n \geq 0}$ is a sequence of integers such that

$$\tau(n) \leq n - 1, \quad \forall n \in \mathbb{N}_0, \quad \text{and} \quad \lim_{n \rightarrow \infty} \tau(n) = \infty \quad (1.1)$$