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COERCIVE ESTIMATES AND SEPARATION FOR PARTIAL DIFFERENTIAL OPERATORS IN HILBERT SPACES ASSOCIATED WITH THE EXISTENCE AND UNIQUENESS THEOREM

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Abstract. In this paper, we have studied the coercive estimate and separation for the following partial differential operator

$$Au(x) = \Delta\left(a(x)\Delta u(x)\right) - b(x)\Delta u(x) + V(x)u(x)$$

for all $x \in R^n$, in the Hilbert space $H = L_2(R^n, H_1)$ with the operator potential $V(x) \in C^2(R^n, L(H_1))$, where $a(x) \geq 0$, $b(x) \geq 0$ are real - valued continuous functions and $L(H_1)$ is the space of all bounded linear operators on the Hilbert space H_1 and $\Delta\Delta u(x)$ is the biharmonic differential operator, while $\Delta u(x) = \sum_{i=1}^{n} \frac{\partial^2 u(x)}{\partial x_i^2}$ is the Laplace operator in R^n. Moreover, we have studied the existence and uniqueness of the solution for the non-homogeneous partial differential equation

$$Au(x) = \Delta\left(a(x)\Delta u(x)\right) - b(x)\Delta u(x) + V(x)u(x) = f(x),$$

in the Hilbert space H as an application of the separation approach, where $f(x) \in H$.

Keywords. Separation, differential operator, coercive estimate, Hilbert space $H = L_2(R^n, H_1)$, operator potential, existence and uniqueness theorem.

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1 Introduction

The concept of separation for differential operators was first introduced by Everitt and Giertz [10,11]. They have obtained the separation results for the Sturm Liouville differential operator

$$Au(x) = -u''(x) + V(x)u(x), \; x \in R, \tag{1}$$

in the space $L_2(R)$. They have studied the following question: What are the conditions on $V(x)$ such that if $u(x) \in L_2(R)$ and $Au(x) \in L_2(R)$ imply that