

ON THE GENERALIZED MONTGOMERY IDENTITY FOR DOUBLE INTEGRALS

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Abstract. In this paper, we establish a generalized Montgomery identity for double integrals and some new generalized Ostrowski type inequality for double integrals is obtained by using fairly elementary analysis.

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1 Introduction

In 1938, the classical integral inequality established by Ostrowski [9] as follows:

Theorem 1. Let $f : [a, b] \rightarrow \mathbb{R}$ be a differentiable mapping on (a, b) whose derivative $f' : (a, b) \rightarrow \mathbb{R}$ is bounded on (a, b) , i.e., $\|f'\|_\infty = \sup_{t \in (a, b)} |f'(t)| < \infty$. Then, the inequality holds:

$$\left| f(x) - \frac{1}{b-a} \int_a^b f(t) dt \right| \leq \left[\frac{1}{4} + \frac{(x - \frac{a+b}{2})^2}{(b-a)^2} \right] (b-a) \|f'\|_\infty \quad (1)$$

for all $x \in [a, b]$. The constant $\frac{1}{4}$ is the best possible.

Inequality (1) has wide applications in numerical analysis and in the theory of some special means; estimating error bounds for some special means, some mid-point, trapezoid and Simpson rules and quadrature rules, etc. Hence inequality (1) has attracted considerable attention and interest from mathematicians and researchers. Due to this, over the years, the interested reader is also referred to ([1]-[8],[13]-[21]) for integral inequalities in several