

NEUTRAL FUNCTIONAL FRACTIONAL DIFFERENTIAL INCLUSIONS WITH IMPULSES AT VARIABLE TIMES

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Abstract. In this work, we investigate the existence of solutions for a class of initial value problems of fractional-order impulsive neutral functional differential inclusions with variable moments.

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1 Introduction

This article is concerned with the existence of solutions to the following initial value problem (IVP) of impulsive fractional-order neutral differential inclusions with variable times:

$${}^C D^\alpha [u(t) - g(t, u_t)] \in F(t, u_t), \text{ a.e. } t \in J, t \neq \tau_k(u(t)), \quad (1)$$

$$u(t^+) = I_k(u(t)), t = \tau_k(u(t)), k = 1, 2, \dots, p, \quad (2)$$

$$u(t) = \phi(t), t \in [-r, 0], 0 < r < \infty, \quad (3)$$

where $0 < \alpha \leq 1$, ${}^C D^\alpha$ is the Caputo fractional derivative, $J = [0, T]$, $F : J \times \mathcal{Z} \rightarrow \mathcal{P}(R)$ is compact convex valued multivalued map ($\mathcal{P}(R)$ is the family of all non-empty subsets of R), $\mathcal{Z} = \{\Phi : [-r, 0] \rightarrow R \text{ is continuous with the exception of a finite number of points } s \text{ where } \Phi(s^-) \text{ and } \Phi(s^+) \text{ exist with } \Phi(s^-) = \Phi(s)\}$, and $g : J \times \mathcal{Z} \rightarrow R$, $I_k : R \rightarrow R$ and $\tau_k : R \rightarrow R$, $k = 1, 2, \dots, p$ are given functions satisfying certain assumptions to be specified later. For any function u defined on $[-r, T]$ and any $t \in J$, we denote by u_t the element of \mathcal{Z} defined by $u_t = u(t + \theta)$, $\theta \in [-r, 0]$.

The topic of fractional-order impulsive differential equations and inclusions has recently been addressed by many researchers as this branch of