

OSTROWSKI TYPE INEQUALITIES FOR HARMONICALLY s -CONVEX FUNCTIONS VIA FRACTIONAL INTEGRALS

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Abstract. In this paper, a new identity for fractional integrals is established. Then by making use of the established identity, some new Ostrowski type inequalities for harmonically s -convex functions via Riemann–Liouville fractional integral are obtained.

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1 Introduction

Let $f : I \rightarrow \mathbb{R}$, where $I \subseteq \mathbb{R}$ is an interval, be a mapping differentiable in I° (the interior of I) and let $a, b \in I^\circ$ with $a < b$. If $|f'(x)| \leq M$, for all $x \in [a, b]$, then the following inequality holds

$$\left| f(x) - \frac{1}{b-a} \int_a^b f(t) dt \right| \leq M(b-a) \left[\frac{1}{4} + \frac{(x - \frac{a+b}{2})^2}{(b-a)^2} \right] \quad (1)$$

for all $x \in [a, b]$. This inequality is known in the literature as the Ostrowski inequality (see [17]), which gives an upper bound for the approximation of the integral average $\frac{1}{b-a} \int_a^b f(t) dt$ by the value $f(x)$ at point $x \in [a, b]$. For some results which generalize, improve and extend the inequalities(1) we refer the reader to the recent papers (see [1, 7, 16]).

In [6], Hudzik and Maligranda considered the following class of functions:

Definition 1 A function $f : I \subseteq \mathbb{R}_+ \rightarrow \mathbb{R}$ where $\mathbb{R}_+ = [0, \infty)$, is said to be s -convex in the second sense if

$$f(\alpha x + \beta y) \leq \alpha^s f(x) + \beta^s f(y)$$

for all $x, y \in I$ and $\alpha, \beta \geq 0$ with $\alpha + \beta = 1$ and s fixed in $(0, 1]$. They denoted this class of by K_s^2 .