Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **24 (2017)** 127-138

PERIODIC SOLUTIONS FOR SECOND ORDER DIFFERENCE EQUATIONS WITH DOUBLE RESONANCE VIA COMPUTATIONS OF THE CRITICAL GROUPS

Xiaosheng Zhang[1], Hongting Shi[1], Yi Zhang[2], Fang Wang[3]
[1]*School of Mathematical Sciences*,
Capital Normal University, Beijing, 100048, China

[2]*College of Science*, *Department of Mathematics*
China University of Petroleum-Beijing, 102249, China

[3]Beijing Chen Jinglun High School, Beijing, 100020, China

Abstract. The existence of multiple periodic solutions for a class of difference equations with double resonance is studied via variational methods and the Morse theory. By relaxing the crucial conditions used in the literature, the results here improve the known results.

Keywords. difference equations; periodic solutions; critical groups.

1 Introduction

For a given positive integer p, consider the following periodic problem on difference equation

$$\begin{cases} -\Delta^2 x(k-1) = f(k, x(k)), \\ x(k+p) = x(k), \end{cases} \qquad k \in \mathbf{Z} \equiv \{0, \pm 1, \pm 2, \cdots\}, \tag{1.1}$$

where Δ is the forward difference operator defined by $\Delta x(k) = x(k+1) - x(k)$ and $\Delta^2 x(k) = \Delta(\Delta x(k))$ for $k \in \mathbf{Z}$. Throughout this paper, we always assume that

(f_1) $f : \mathbf{Z} \times \mathbf{R} \to \mathbf{R}$ is C^1-differentiable with respect to the second variable and satisfies $f(k+p, t) = f(k, t)$ for $(k,\ t) \in \mathbf{Z} \times \mathbf{R}$ and $f(k, 0) \equiv 0$ for $k \in \mathbf{Z}$.

It is well known that in different fields of research, such as computer science, mechanical engineering, control systems, artificial or biological neural networks, economics and many others, the mathematically modeling of important questions leads naturally to the consideration of nonlinear difference equations. As a natural phenomenon, resonance exists in the real