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APPROXIMATION PROPERTIES OF q-BARNSTEIN TYPE OPERATORS

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Abstract. In the present paper, we introduce the q-analogue of Bernstein operators with shifted knots. First, we prove the basic convergence of the introduced operators and then obtain the rate of convergence by these operators in terms of the modulus of continuity and prove a Voronovskaja type theorem. Further, we study local approximation result for the said operators. We also show comparisons by some illustrative graphics for the convergence of operators with the help of Matlab.

Keywords. q-Bernstein type operators; rate of convergence; modulus of continuity; Voronovskaja type theorem; K-functional; local approximation.

AMS (MOS) subject classification: 41A10, 41A25, 41A36.

1 Introduction

The applications of q-calculus in the area of approximation theory were initiated by Lupas [11], who first introduced q-Bernstein polynomials. Also in the last decade, Phillips [16] proposed other q-Bernstein polynomials, which became popular. Later several researchers obtained the interesting properties of q-Bernstein polynomials. After that several researchers have estimated the approximation properties of several operators. In the recent years, many researchers have studied the approximation properties for linear positive operators [1, 3, 12, 13]. Mursaleen et al in [14, 15] has also obtained statistical approximation properties for new positive linear operators and some approximation theorems for generalized q-Bernstein-Schurer operators.

Let $C[0,1]$ denote the set of all continuous functions on $[0,1]$ which is equipped with sup-norm $\|.\|_{C[0,1]}$. In [4], Bernstein introduced the following well-known positive linear operators

$$B_n(f;x) = \sum_{k=0}^{n} \binom{n}{k} x^k (1-x)^{n-k} f\left(\frac{k}{n}\right) \tag{1.1}$$

and he showed that if $f \in C[0,1]$, then $B_n(f;x) \rightrightarrows f(x)$ where " $\rightrightarrows$ "