

CLASSIFICATIONS OF SOLUTIONS OF SECOND ORDER NONLINEAR NEUTRAL DIFFERENCE EQUATIONS OF MIXED TYPE

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Abstract. In this paper, we classify all solutions of the second order nonlinear neutral difference equation of mixed type of the form

$$\Delta (a_n \Delta (x_n + bx_{n-\tau_1} + cx_{n+\tau_2})) + p_n x_{n+1-\sigma_1}^\alpha + q_n x_{n+1+\sigma_2}^\beta = 0, \quad n \geq n_0 > 0$$

into four disjoint classes, and derive some sufficient conditions for the existence/nonexistence of solutions in these classes. Examples are provided to illustrate the results. The results presented here generalize and extend some of the known results.

Keywords. Oscillation, second order, neutral difference equation, mixed type.

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1 Introduction

This paper is concerned with the second order nonlinear neutral difference equation of mixed type of the form

$$\Delta (a_n \Delta (x_n + bx_{n-\tau_1} + cx_{n+\tau_2})) + p_n x_{n+1-\sigma_1}^\alpha + q_n x_{n+1+\sigma_2}^\beta = 0, \quad n \geq n_0 > 0, \quad (1.1)$$

subject to the following conditions:

(C₁) $\{a_n\}$ is a sequence of positive real numbers for all $n \geq n_0$;

(C₂) $\{p_n\}$ and $\{q_n\}$ are sequences of real numbers;

(C₃) b and c are arbitrary constants;

(C₄) τ_1 , τ_2 , σ_1 and σ_2 are nonnegative integers;

(C₅) α and β are ratios of odd positive integers.