Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **23 (2016)** 399-427

EXISTENCE OF AN INFINITE-HORIZON OPTIMAL IMPULSE CONSUMPTION OF A GEOMETRIC BROWNIAN MOTION WITH VARIABLE COEFFICIENTS

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Abstract. We consider the problem of maximizing expected lifetime utility from consumption of a geometric Brownian motion with variable coefficients, in the presence of controlling costs with a fixed component. Under general assumptions on the utility function and the intervention costs our main result is to show that, if the discount rate is large enough, there always exists an optimal Markovian impulse control for this problem. We compute explicitly the optimal consumption in the case of constant coefficients of the process, linear utility and a two values discount rate. In this illustrative example the value function is not C^1 and the verification theorems commonly used to characterize the optimal control cannot be applied.

Keywords. Existence of optimal feedback controls; impulse control; quasi variational inequalities; geometric Brownian motion with variable coefficients; intervention costs with a fixed component.

AMS (MOS) subject classification: 49J40, 49L20, 49N25, 60G40, 60J60.

1 Introduction

In this paper we consider the optimal consumption of a diffusion process S_t, which is a geometric Brownian motion generalized to have variable coefficients. The evolution of S_t, in absence of control, is described by the Itô's stochastic differential equation

$$dS_t = S_t\mu(S_t)dt + S_t\sigma(S_t)dB_t \tag{1}$$

where B_t is a one-dimensional Brownian motion and the functions μ and σ are assumed to be bounded and Lipschitz continuous. Like the standard geometric Brownian motion, when μ, σ are constant, the process remains positive for all $t > 0$, if $S_0 > 0$. The agent wants to maximize the expected utility from consumption of S over an infinite horizon. Consumption is possible at any time but whenever S is consumed some quantity of S is lost as an