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IDE VECTOR CALCULUS: AN INTRODUCTION

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Abstract

IDEs have been discussed in several previous papers [1,2,3,4,5,6]. A recent paper began the outline of a *Theory of IDEs* [6]. However, introducing a new field of mathematics requires answering two questions. (1) why is it useful; (2) how does it fit into the existing body of mathematics? The usefulness of IDEs has been established in the field of neuroscience where improvements of 100:1 in computational efficiency have been achieved [7]. In this paper I will begin to show how a theory of IDEs fits into the greater body of mathematics, especially applied mathematics.

It is essential for the theory of IDEs to have familiar and conventional engineering computational tools that do not rely on an understanding of the abstract mathematics of dynamics. It is also essential that the tools of engineering be connected back to the fundamental theory of complexity in dynamics, i.e., the Smale Birkhoff theorem [8]. Therefore, in this paper we examine the use of gradient, divergence and curl in analyzing the dynamics of the scroll.

As noted in previous papers, it is the nonlinearity of the divergence that is an indicator of the potential for complex dynamics due to the stretching factor that is provided by the nonlinear divergence. The same statement can be made about the nonlinear curl.

In this paper I will examine the use of the nonlinear divergence and circulation component of the curl to study the emergence of complexity. The baseline system will be a simple linear system having a hyperbolic fixed point at the origin. The stable manifold will be the $z-$axis and the unstable manifold will be the $x-y$ plane. The baseline system will have positive divergence and the curl ($2\,\omega$) will be relatively large and will be produce the spiral source in the $x-y$ plane.

A very simple relatively very low frequency ($\gamma = .001$) nonlinear sinusoidal forcing will be used to perturb the linear system. The ratio of the frequency of the linear system ($\omega = 5.0$) to the nonlinear forcing ($\gamma = 0.001$) will be 5000:1. The amplitude of the sinusoidal forcing function will be increased until the emergence of complexity.

Keywords. Chaos, Algebra, complexity, dynamical systems, dynamical synthesis.

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1 Introduction

Infinitesimal Diffeomorphism Equations (IDE) have been discussed in several papers [1,2,3,4,5,6]. IDEs are a special form of FDEs. However, unlike a typical FDE, an IDE produces a time series that more closely resembles the solution of an ODE without necessarily being a solution of an ODE. IDEs are not derived from the laws of physics but rather from the laws of complexity: stretching and folding. In this