SOLUTIONS OF FIRST-ORDER PERIODIC BOUNDARY VALUE PROBLEM IN GENERALIZED METRIC SPACES

Hemant Kumar Nashine[1] Ravi P. Agarwal[1] and Anita Gupta[2]

[1] Department of Mathematics
Texas A & M University - Kingsville - 78363-8202, Texas, USA
Email: drhknashine@gmail.com, agarwal@tamuk.edu

[2]Department of Mathematics
C.V.Raman University, Bilaspur, Chhattisgarh, India
Email: gupta.archu123@rediffmail.com

Abstract. This article investigates the existence of a solution of first-order periodic boundary value problem

$$\begin{cases} x'(t) = f(t, x(t)), & t \in I = [0, T] \\ x(0) = x(T), \end{cases}$$

where $T > 0$ and $f : I \times \mathbb{R} \to \mathbb{R}$ is a continuous function, in a space where self-distance of a point may be non-zero. To accomplish this goal, we launch a new α-weakly contractive mapping which involve rational terms and then build up fixed point results for weakly α-admissible mapping. To illustrate our results, we give throughout the paper some examples. Finally, we suggest some future work to find positive solutions of three classes of nonlinear fractional differential equations.

Keywords. fixed point; b-metric-like space; altering distance function; weakly α-admissible map, boundary-value problem

AMS (MOS) subject classification: 47H10; 54H25; 34C25.

1 Introduction and preliminaries

The Banach Fixed Point Theorem is a source of inspiration for past and present researchers of mathematics and different branches of science and technology. A huge amount of literature is witnessed on applications, generalizations and extensions of this principle carried out by several authors in different directions, e.g., by weakening the hypothesis, using different setups, considering different mappings and generalized form of metric spaces.

The study of new classes of spaces in mathematics and their basic properties are always favorite topics of interest among the mathematical research community. Recently, some authors have introduced some generalizations of metric spaces in several ways and have studied fixed point theory and its applications. In this context, Matthews [9] introduced the notion of a partial