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EXISTENCE OF BOUND STATES FOR $(N+1)$-COUPLED LONG-WAVE–SHORT-WAVE INTERACTION EQUATIONS

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Abstract. We prove the existence of an infinite family of smooth positive bound states for $(N+1)$-coupled long-wave–short-wave interaction equations. The system describes the interaction between N short waves and a long wave and is of interest in physics and fluid dynamics. The existence result is obtained using the variational technique based on the concentration compactness principle.

Keywords. nonlinear Schrödinger-type equation ; Korteweg de Vries-type equation ; long-wave–short-wave interaction ; bound-state solutions ; existence

AMS (MOS) subject classification: 35Q53, 35Q55, 35A15, 35B35.

1 Introduction

A non-linear system of interaction between a complex short-wave field u and a real long-wave field v of the form

$$\begin{cases} i\partial_t u + \partial_x^2 u = \alpha uv + \beta |u|^2 u \\ \partial_t v + \partial_x^3 v + v\partial_x v = \gamma \partial_x \left(|u|^2\right), \end{cases} \tag{1.1}$$

was first studied in [24] concerning the well-posedness of the Cauchy problem. The system (1.1) which has an interaction between a nonlinear Schrödinger (NLS)-type short wave and a Korteweg de Vries (KdV)-type long wave appears in a wide variety of physical systems. The reader may refer to [3] for a general theory of the non-linear long-wave–short-wave interaction (LSI) model. Numerous successful applications of the LSI model exist in different contexts of fluid dynamics such as capillary-gravity waves in [17], sonic-Langmuir solitons in [16, 25], Alfvén waves in [23], and Bose-Einstein condensates in [20], to mention but a few.

Let us consider the multicomponent LSI system

$$\begin{cases} i\partial_t u_j + \partial_x^2 u_j = \alpha_j u_j v + \beta_j |u_j|^2 u_j, \ j = 1, 2, \dots N, \\ \partial_t v + \partial_x^3 v + v\partial_x v = \dfrac{1}{2}\alpha_j \ \partial_x \left(\displaystyle\sum_{j=1}^N |u_j|^2 \right), \end{cases} \tag{1.2}$$