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# OPTIMAL CONTROL WITH FINAL OBSERVATION OF A FRACTIONAL DIFFUSION WAVE EQUATION

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**Abstract.** We consider a controlled fractional diffusion wave equation involving Riemann-Liouville fractional derivative or order $\alpha \in (3/2,2)$. First we prove by means of eigenfunction expansions the existence of solutions to such equations. Then we show that we can approach the fractional integral of order $2-\alpha$ of the state at final time by a desired state by acting on the control. Using the first order Euler-Lagrange optimality, we obtain the characterization of the optimal control.

**Keywords.** Riemann-Liouville fractional derivative; Caputo fractional derivative; Initial value /boundary value problem.

**AMS (MOS) subject classification:** 49J20, 49K20, 26A33.

# 1 Introduction

Let $n \in \mathbb{N}^*$ and $\Omega$ be a bounded open subset of $\mathbb{R}^n$ with boundary $\partial\Omega$ of class $\mathcal{C}^2$. For the time $T > 0$, we set $Q = \Omega\times]0,T[$ and $\Sigma = \partial\Omega\times]0,T[$, and we consider the following fractional diffusion wave equation:

$$\left\{\begin{array}{rcll} D^{\alpha}_{RL}y(x,t) - \Delta y(x,t) &=& v(x,t) & (x,t) \in Q \\ y(\sigma,t) &=& 0 & (\sigma,t) \in \Sigma \\ I^{2-\alpha}y(x,0^+) &=& y^0 & x \in \Omega \\ \dfrac{d}{dt}I^{2-\alpha}y(x,0^+) &=& y^1 & x \in \Omega \end{array}\right. \tag{1}$$

where $1 < \alpha < 2$, $y^0 \in H^2(\Omega)\cap H^1_0(\Omega)$, $y^1 \in L^2(\Omega)$ and $v \in L^2(Q)$. $I^{2-\alpha}y(x,0^+) = \lim\limits_{t\to 0} I^{2-\alpha}y(x,t)$ and $\dfrac{d}{dt}I^{2-\alpha}y(x,0^+) = \lim\limits_{t\to 0}\dfrac{d}{dt}I^{2-\alpha}y(x,t)$ where the fractional integral $I^{\alpha}$ of order $\alpha$ and the fractional derivative $D^{\alpha}_{RL}$ of order $\alpha$ are to be understood in the Riemann-Liouville sense.

There are many works on fractional diffusion wave equation. For instance, Mainardi et al. [18, 19] generalized the diffusion equation by replacing the first