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LOCAL EXISTENCE TO THE NAVIER-STOKES EQUATIONS THROUGH A REVERSELY-DIFFUSIVE UPPER SOLUTION

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Abstract. A local existence result is demonstrated for the Navier-Stokes equations in all of $\mathbb{R}^3$. Unlike other local existence results, the method of proof is elementary in that it does not use weak solutions. Solutions are shown to exist in Hölder-continuous function spaces by the use of the contraction mapping principle. The gradient of the pressure term in the Navier-Stokes equations is expressed in terms of the velocity. By a symmetry transformation, the problem is transformed into an equivalent one having a very small initial velocity. A specific upper solution to the heat equation which mimics a reverse-diffusion process enables the implemetation of the contraction mapping principle.

Keywords. Navier-Stokes equations, nonlinear parabolic equations, local existence theorems.

AMS (MOS) Subject classification: 35K45, 35K55, 35Q30, 76D05

1 Introduction

This article presents an elementary proof of the existence of a local solution in time to the homogeneous incompressible Navier-Stokes equations defined on the whole Euclidean space $\mathbb{R}^3$.

More precisely, the following problem for the Navier-Stokes equations is considered

$$\begin{cases} -\dfrac{\partial u}{\partial t} + \nu\Delta u - (u\cdot\nabla)u - \nabla p = 0 & \text{in } \mathbb{R}^3\times[0,T) \\ \nabla\cdot u = 0 & \text{in } \mathbb{R}^3\times[0,T) \\ u(x,0) = u_0(x) & \text{in } \mathbb{R}^3 \end{cases} \tag{1}$$

where $\nu > 0$ denotes the constant kinematic viscosity and where the initial