

DIFFEOMORPHISMS WITH AVERAGE AND ASYMPTOTIC AVERAGE SHADOWING

Manseob Lee¹ and Junmi Park²

¹Department of Mathematics, Mokwon University,
Daejeon, 302-729, Korea.

²Department of Mathematics, Chungnam National University,
Daejeon, 305-764, Korea.

E-mail: lmsds@mokwon.ac.kr, pjmnds@cnu.ac.kr

Abstract. Let $f : M \rightarrow M$ be a diffeomorphism on a closed smooth manifold M , and let p, q be a hyperbolic periodic points of f . In this paper, we show that (i) if f has the C^1 -robustly average, asymptotic average shadowing property and $p \sim q$ then it is transitive Anosov, and (ii) C^1 -generically, if f has the average, asymptotic average shadowing property and $W^s(p) \cap W^u(q) \neq \emptyset$ then it is transitive Anosov.

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1 Introduction

The dynamical system is mostly studied by the behavior of the orbits of a given system. In fact, the structure of the orbit of the dynamical system is related to the shadowing theory. It is a very useful notion in the dynamical systems (see [21]). Actually, Sakai showed in [25] that if a diffeomorphism belongs to the C^1 -interior of the set of all diffeomorphisms having the shadowing property then it satisfies the structural stability. Robinson showed in [23] that if a system is structurally stable then it has the shadowing property. It is well known that even if a dynamical system satisfies the structural stability, the system is not Anosov. In fact, if a dynamical system is structurally stable then the system satisfies Axiom A and the strong transversality condition. If a system satisfies Axiom A then the nonwandering set is hyperbolic and it is the closure of the set of all periodic points. Then the nonwandering set has the finite closed, invariant and transitive sets and such the closed set is called a *basic set*. The following is still an open problem: if a system is Anosov then is it transitive? Franks [6] and Newhouse [20] proved that if a system is codimension one Anosov then it is transitive. It is well-known that if the nonwandering set is just one basic set then it is transitive Anosov. In