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POSITIVE SOLUTIONS FOR A CLASS OF MULTIPARAMETER ELLIPTIC SYSTEMS

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Abstract. We consider an elliptic system of the form

$$\left.\begin{array}{rcl} -\Delta_p u = \lambda_1 f_1(u) + \mu_1 \frac{g_1(v)}{v^{\alpha_1}} & \text{in} & \Omega; \\ -\Delta_q v = \lambda_2 \frac{f_2(u)}{u^{\alpha_2}} + \mu_2 g_2(v) & in & \Omega; \\ u = v = 0 & \text{on} & \partial\Omega, \end{array}\right\}$$

where $p, q > 1$, $\Delta_m w := |\nabla w|^{m-2}\nabla w$ is the m-Laplacian operator for $m > 1$, $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary, $\lambda_i, \mu_i > 0$ are parameters and $0 \leq \alpha_i < 1$ are fixed constants for $i = 1, 2$. The nonlinearities f_i, $g_i : [0, \infty) \to \mathbb{R}$ are continuous functions satisfying certain p, q-sublinear or combined sublinear conditions at infinity. When $g_1(0) < 0$ and $f_2(0) < 0$, we discuss existence of a positive solution for $\lambda_i + \mu_i >> 1$ for $i = 1, 2$. We also discuss a multiplicity result when $\alpha_1 = 0 = \alpha_2$. Method of sub- and supersolutions are employed to establish these results.

Keywords. Elliptic systems, p-Laplacian, sublinear, multiparameter, positive solutions, infinite semipositone, existence, multiplicity

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1 Introduction

We consider an elliptic system of the form

$$\left.\begin{array}{rcl} -\Delta_p u = \lambda_1 f_1(u) + \mu_1 \frac{g_1(v)}{v^{\alpha_1}} & \text{in} & \Omega; \\ -\Delta_q v = \lambda_2 \frac{f_2(u)}{u^{\alpha_2}} + \mu_2 g_2(v) & \text{in} & \Omega; \\ u = v = 0 & \text{on} & \partial\Omega, \end{array}\right\} \quad (1.1)$$

where $\Delta_m w := |\nabla w|^{m-2}\nabla w$ is the m-Laplacian operator for $m > 1$ and $\Omega \subset \mathbb{R}^N$; $N \geq 1$, is a bounded domain with smooth boundary (a bounded interval if $N = 1$). For $i = 1, 2$, $0 \leq \alpha_i < 1$ are fixed constants and $\lambda_i, \mu_i > 0$ are parameters.

The nonlinearities f_i, $g_i : [0, \infty) \to \mathbb{R}$ are continuous functions such that $g_1(0) < 0$ and $f_2(0) < 0$. Let $\tilde{g}_1(s) := \frac{g_1(s)}{s^{\alpha_1}}$ and $\tilde{f}_2(s) := \frac{f_2(s)}{s^{\alpha_2}}$. We make the following assumptions: