Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **23 (2016)** 263-272
 http://www.watam.org

BOUNDARY VALUE PROBLEM FOR A THREE-TIME-SCALE SINGULARLY PERTURBED DISCRETE SYSTEM

Tahia Zerizer

Mathematics Department, College of Science,
Jazan University, Jazan, Kingdom of Saudi arabia.

tzerizer@yahoo.fr, tzerizer@gmail.com

Abstract. In this paper, we consider a three time scale state model arising in the open-loop optimal control of singularly perturbed discrete systems. The *perturbation method* is described to solve a two point boundary value problem exhibiting boundary layer behavior at the initial and final points. We give conditions that guarantee existence and uniqueness of the solution, we achieve model order reduction and remove the time scale. A convergent iterative algorithm is provided to compute asymptotic approximations without need to use boundary layer correction terms as for the *singular perturbation method.*

Keywords. Singularly perturbed system, Discrete-time system, Multitime-scale system, Time scale separation, Reduced order system, Boundary value problem.

AMS (MOS) subject classification: 93C05, 93C55, 93C70, 93C73.

1 Introduction

This paper is intended for engineers and applied mathematicians concerned with model-order reduction, separation of time scales and simplified procedures for discrete control systems analysis. On this topic, there have been many related research on the analysis of singularly perturbed discrete-time systems, see [1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 16]. In this work we examine a time-varying three-time-scale state model arising in the open-loop optimal control of singularly perturbed discrete systems. More precisely, we consider the system:

$$\begin{pmatrix} x_{k+1} \\ y_{k+1} \\ \varepsilon z_{k+1} \end{pmatrix} = \begin{pmatrix} A_{11}(k) & \varepsilon A_{12}(k) & A_{13}(k) \\ A_{21}(k) & \varepsilon A_{22}(k) & A_{23}(k) \\ A_{31}(k) & \varepsilon A_{32}(k) & A_{33}(k) \end{pmatrix} \begin{pmatrix} x_k \\ y_k \\ z_k \end{pmatrix}, \quad k = 0, \dots, N-1, \tag{1.1}$$

where $x_k \in \Re^n$, $y_k \in \Re^m$ and $z_k \in \Re^p$ are the state vectors at the kth discrete time; $A_{ij}(k)$, $i, j = 1, 2, 3$, $k = 0, \dots, N-1$, are constant matrices with appropriate dimensions, and ε is a small real parameter. We associate