BOUNDARY VALUE PROBLEM FOR A THREE-TIME-SCALE SINGULARLY PERTURBED DISCRETE SYSTEM

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Abstract. In this paper, we consider a three time scale state model arising in the open-loop optimal control of singularly perturbed discrete systems. The perturbation method is described to solve a two point boundary value problem exhibiting boundary layer behavior at the initial and final points. We give conditions that guarantee existence and uniqueness of the solution, we achieve model order reduction and remove the time scale. A convergent iterative algorithm is provided to compute asymptotic approximations without need to use boundary layer correction terms as for the singular perturbation method.

Keywords. Singularly perturbed system, Discrete-time system, Multitime-scale system, Time scale separation, Reduced order system, Boundary value problem.

AMS (MOS) subject classification: 93C05, 93C55, 93C70, 93C73.

1 Introduction

This paper is intended for engineers and applied mathematicians concerned with model-order reduction, separation of time scales and simplified procedures for discrete control systems analysis. On this topic, there have been many related research on the analysis of singularly perturbed discrete-time systems, see [1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 16]. In this work we examine a time-varying three-time-scale state model arising in the open-loop optimal control of singularly perturbed discrete systems. More precisely, we consider the system:

\[
\begin{pmatrix}
  x_{k+1} \\
  y_{k+1} \\
  z_{k+1}
\end{pmatrix}
= \begin{pmatrix}
  A_{11}(k) & \varepsilon A_{12}(k) & A_{13}(k) \\
  A_{21}(k) & \varepsilon A_{22}(k) & A_{23}(k) \\
  A_{31}(k) & \varepsilon A_{32}(k) & A_{33}(k)
\end{pmatrix}
\begin{pmatrix}
  x_k \\
  y_k \\
  z_k
\end{pmatrix}, \quad k = 0, \ldots, N-1, \quad (1.1)
\]

where \( x_k \in \mathbb{R}^n \), \( y_k \in \mathbb{R}^m \) and \( z_k \in \mathbb{R}^p \) are the state vectors at the \( k \)th discrete time; \( A_{ij}(k) \), \( i, j = 1, 2, 3 \), \( k = 0, \ldots, N-1 \), are constant matrices with appropriate dimensions, and \( \varepsilon \) is a small real parameter. We associate