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The Hirsch Conjecture

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Abstract. "A major challenge to mathematicians is to determine which dynamical systems are chaotic and which are not. Ideally one should be able to tell from the form of the differential equation" – Morris W. Hirsch, 1985 [1].

Since Hirsch set forth his conjecture in 1985, two important issues have come to light. Issue 1: In the Smale-Birkhoff Theorem [2], the horseshoe paradigm was introduced without examining how much stretching combined with how much folding produces a transverse homoclinic point. Also, the paradigm does not shed any light on the interrelationship between stretching and folding. For example, implicitly, the paradigm requires a 180 degree rotation as the folding dynamic. As a result, it is difficult to relate the theorem to specific transformations such as the Hénon map. Issue 2: How are stretching and folding to be recognized in the "form" of a differential equation or transformation?

This letter addresses these two issues and provides some answers.

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1 Introduction

Hirsch set forth his 1985 conjecture. Addressing this conjecture has proven to be a formidable task. The reason is made clear by the following two equations Eq.(1) and Eq.(2):

$$\ddot{x} + \dot{x} + x^3 = \cos(t) \tag{1}$$

$$\ddot{x} + 0.05\dot{x} + x^3 = 7.5\cos(t) \tag{2}$$

As is well known, Eq.(2) has a strange attractor that outlines the unstable manifold of a hyperbolic fixed point, whereas Eq.(1) "appears" not to be capable of producing strange attractor from any initial condition. But these two equations would "seem" to have the same form. [1] The Hirsch Conjecture is difficult to address because it may not be exactly clear as to what constitutes the "form" of an equation. Typically, we would refer to the equation's algebraic form as its "form". In the case of these two equations we have two

[1] Note that the damping factor only serves to make chaos visible using numerical methods. It is not an essential factor in creating a transverse homoclinic point.