

APPROXIMATION OF OPTIMAL PRICES WHEN BASIC DATA ARE WEAKLY DEPENDENT

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Abstract. Corresponding to the Black-Scholes stochastic differential equation, Yoshihara (2012) introduced a difference equation based on weakly dependent stationary random variables and proved that its solution converges almost surely to a geometric Brownian motion with an annual drift parameter and a volatility which come from the assumption on the random variables. In this paper, we show some further results and present their applications by using approximations of some optimal prices in the Black-Scholes market.

Keywords. Difference equation, Weakly dependent random variable, Black-Scholes type stochastic differential equation, stationary sequence, Wiener process.

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1 Introduction

In the mathematical finance model, the specific problem is finding the optimal time to sell a stock as mentioned in [1] and [2]. In the Black-Scholes model, its stock price Y_t is given by the solution of the following stochastic differential equation driven by a standard one-dimensional Brownian motion:

$$dY_t = \mu Y_t dt + \kappa Y_t dB_t, \quad (1)$$

where $\mu \in \mathbf{R}$ and $\kappa > 0$. To consider the optimal stopping problems, discrete finance models are actual; however, the problem is not easy to solve for models described by random walks. In this paper, we consider such problems using methods of strong approximations of Brownian motion. In [12] and [13], Yoshihara considered approximations of the solution of the Black-Scholes model based on weakly dependent stationary random variables by the Euler-Maruyama scheme. For the discrete models, we can use properties of Brownian motions to consider the optimal stopping problems.