Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **23 (2016)** 155-193

A TOPOLOGY OPTIMIZATION APPROACH FOR MICRO-ARCHITECTURED SYSTEMS ON SINGULARLY PERTURBED PERIODIC MANIFOLDS–TWO-SCALE ASYMPTOTIC ANALYSIS AND THE INFLUENCE OF THE NETWORK TOPOLOGY

Erik Kropat[1] and Silja Meyer-Nieberg[2] and Gerhard-Wilhelm Weber[3]*

[1]Institute for Applied Computer Science
University of the Bundeswehr Munich, Werner-Heisenberg-Weg 39,
85577 Neubiberg, Germany

[2]Institute for Theoretical Computer Science, Mathematics and Operations Research
University of the Bundeswehr Munich, Werner-Heisenberg-Weg 39,
85577 Neubiberg, Germany

[3]Institute of Applied Mathematics
Middle East Technical University, 06531 Ankara, Turkey

Abstract. Micro-architectured systems and materials on very large periodic networks arise in many applications ranging from engineering sciences to multi-scale physics. The corresponding microscopic models are affected by an inherent interdependence of the physical process on the network and the topology of the underlying periodic manifold. In particular, the global characteristics of the micro-architectured systems highly depend on the network's topology. A topology optimization procedure is proposed that aims to improve the geometry of the parameter-dependent periodic graph. In this way, the global characteristics of the system are optimized and a topology with specified set of design goals is selected. Microscopic models are very challenging from an analytical and a numerical perspective. The high number of branches and transition conditions at the ramification nodes as well as the highly oscillating coefficients make such problems difficult to solve. Two-scale asymptotic methods are applied to derive approximating homogenized models.

*Further positions: Faculty of Economics, Business and Law, University of Siegen, Germany (Permanent Guest Professor); School of Science, Information Technology and Engineering, Federation University of Australia, Ballarat, Australia (Adjunct Professor), Center for Research & Development in Mathematics and Applications (CIDMA), Department of Mathematics, University of Aveiro, Portugal (Collaborator); University of North Sumatra (University of Sumatera Utara, USU), Medan, Indonesia (Representative - Advisor)

The coefficients of these macroscopic systems characterize the network model on a global scale. In this paper, we further extend previous approaches and provide an explicit representation of the homogenized coefficients in terms of the network's geometry and the coefficients of the microscopic system. This representation offers new insights in the intrinsic relationship between the underlying topological microstructure and the effective behavior of the system on a global scale. The topology optimization approach combines the homogenization of differential equation on periodic manifolds and the topology optimization of the underlying periodic micro-geometry. We demonstrate the effectiveness of this approach at the example of singularly perturbed diffusion-advection-reaction equations on a parameter-dependent honeycomb structure.

Keywords. Topology optimization, homogenization theory, asymptotic analysis, two-scale method, networks, diffusion-advection-reaction equations, boundary value problems on graphs and networks

AMS (MOS) 34B45, 34E13, 34E05, 34E10

1 Introduction

Micro-architectured systems and materials with a periodic structure arise in many applications in science and technology. Such models have been successfully applied in diverse fields like *photonic crystals* in *nanotechnology* [15, 23, 35] and *transport phenomena* in *soil mechanics* and *oil recovery* [2, 3]. Usually, these systems are affected by two interconnected scales: (i) the *local scale*, which measures the highly oscillating effects of the periodic microstructure and (ii) the *global scale*, that describes the macroscopic behaviour of the system. In particular, the *geometry of the microstructure* takes a significant influence on the *global characteristics* of the system or material under consideration. In this paper, we propose a new approach for a *topology optimization of the micro-architecture of a periodic network* that is based on a *two-scale homogenization approach* for differential equations on *singularly perturbed one-dimensional manifolds.* We consider elliptic differential equations on very large periodic networks with a periodic microstructure. Despite the huge amount of publications on *shape optimization* and *topology optimization*, the optimization of the underlying topology of one-dimensional periodic manifolds in the context of homogenization theory has - to the best of our knowledge - not been discussed in the literature before.

Periodic one-dimensional networks in applications

Differential equations on periodic networks are applied in many disciplines ranging from engineering sciences to multi-scale physics. Because of the periodicity of the underlying physical structure, the system is composed of a huge number of recurrently arranged *prototypical elements.* For example, so-called *porous media approaches* [4, 10] are based on such networks. The pore structures of these media are represented by idealized models in terms of *networks of capillaries.*