

CREATION AND PRESERVATION OF SHOCKS IN LINNIK CONSERVATION LAWS

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Abstract. We consider a Linnik conservation law, which was defined by the Fourier multiplier, [14] and derive some results about its approximate solutions using a numerical method and investigate the solutions if this equations become discontinuous or continuous in large time when the stability parameter (α) is less than 1. Based on the numerical experiments, we discovery the results that the Linnik diffusion produce shocks that do not dissipate over time come from our simulations. That is, the size of the shock is initially decreasing and then the shock begins to recede to the left. We also observe that the speed of the shock movement to the left is negatively proportional to the value of stability parameter.

Keywords. Linnik Lévy Meaure, Linnik diffusions, Lévy stable diffusions, Shocks, Stability parameter.

1 Introduction

Linnik(1953) proved that the function $(1 + |\xi|^\alpha)^{-1}$, $\alpha \in (0, 2]$ is the characteristic function of a symmetric distribution with support \mathbb{R} . In the extreme case $\alpha = 2$, the distribution corresponds to Laplace distribution. Since then, several authors have discussed about the analytic and asymptotic properties, long range dependence and applications of Linnik probability density function, see, e.g.,[2,13,21]. In this article, we are particularly interested to study the creation and preservation of shocks in Linnik conservation laws using a numerical method. Because nonlocal evolution equations driven by α -Linnik laws no general rigorous mathematical results are available.

The subject of linear and nonlinear diffusion equations driven by non-local, pseudo-differential operators witnessed a considerable research activity well represented in both mathematical and physical literature with a wide