PARAMETER ESTIMATED STANDARDIZED U-STATISTICS BY ABSOLUTELY REGULAR SEQUENCES

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Abstract. Extending the results of Gombay and Horváth (1998), we obtain a limit theorem for the maximum of standardized degenerate U-statistics defined by some absolutely regular sequences.

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1 Introduction and result

Let $\{X_i\}$ be independent and identically distributed random variables with a common distribution function $F(x)$ defined on a probability space $(\Omega, \mathbf{F}, P)$. Let $h(x,y)$ be a symmetric function and define

$$U_{k,n} = \sum_{i=1}^{k} \sum_{j=k+1}^{n} h(X_i, X_j) - k(n-k)\theta, \tag{1}$$

where $\theta = Eh(X_1, X_2)$. Horváth and Shao considered a limit distribution for maximally selected standardized U-statistics in the degenerate case ([4]). Let

$$\tilde{h}(x) \ = \ \int (h(x,y) - \theta) dF(y).$$

We assume that $E\tilde{h}(X_1) = 0$. Then, there exist orthogonal eigenfunctions $\{\varphi_j(x); 1 \le j < \infty\}$ and eigenvalues $\{\lambda_j; 1 \le j < \infty\}$ such that

$$\lim_{M \to \infty} \iint \left(h(x,y) - \theta - \sum_{j=1}^{M} \lambda_j \varphi_i(x)\varphi_j(y) \right)^2 \tag{2}$$

$$dF(x)dF(y) \ = \ 0$$