

NUMERICAL APPROACHES TO A STOCHASTIC LOGISTIC MODEL WITH REFLECTION

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Abstract. A stochastic logistic model with reflection at the boundary is considered. Approximation schemes based on the solution of the associated Skorohod problem are introduced, and the results of numerical simulations involving these schemes are presented.

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1 Introduction

The deterministic logistic model for single species of the form

$$\frac{dN}{dt} = r(t)N(t)(K - N(t)), \quad N(0) = N_0 \quad (1)$$

where $r(t)$ is the growth rate and K is the environmental carrying capacity, has been used in mathematical modeling for a long time. By relating the state $N(t)$ to the carrying capacity

$$y(t) = \frac{N(t)}{K}$$

the logistic model is easily transformed to

$$\frac{dy}{dt} = Kr(t)y(t)(1 - y(t)), \quad y(0) = y_0 = \frac{N_0}{K} \quad (2)$$

Under this standardization we see that

$$y(t) \longrightarrow 0 \text{ means } N(t) \longrightarrow 0 \text{ (extinction)}$$

and

$$y(t) \longrightarrow 1 \text{ means } N(t) \longrightarrow K \text{ (saturation).}$$