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NEAR OPTIMALITY AND NEAR EQUILIBRIUM FOR CONTROLLED SYSTEMS WITH WIDEBAND NOISE FOR HYBRID SYSTEMS

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Abstract This work is concerned with near-optimal control of stochastic hybrid systems. To begin, in lieu of the usual setup of dynamics driven by "white" noise of Brownian motion type, we consider systems that are subject to wideband noise disturbances. In addition, there is another Markov chain depicting the random environment that cannot be descried by stochastic differential equations but only representable as a jump process (a continuous-time) Markov chain. Assuming the Markov chain is fast varying, under rather broad conditions, we show that there is a limit system. Using the limit controlled dynamic system as a guidance, we construct controls for the original problem and show that the controls so constructed are near optimal. Using the comparison control techniques, we also examine mean-field type game problems. Finally, we consider the systems that are linear in the continuous states.

Keywords. Near-optimal control, near equilibrium, wideband noise, switching system, mean-field game, weak convergence.

AMS (MOS) subject classification: Primary 93E20, Secondary 60F17, 93C70, 93E03.

1 Introduction

Hybrid systems are featured by the coexistence of continuous dynamics and discrete events and their interactions. They have drawn much needed attentions in recent years; see [21] among others. One of the main reasons is that such systems can be used to better reflect the reality for a wide range of applications in networked systems, communication systems, economic systems, cyber-physical systems, biological, and ecological systems, among others. One class of such hybrid systems is known as switching diffusions. In