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SOME RESULTS ON THE STABILITY AND BIFURCATION OF A DISTRIBUTED DELAY NETWORK

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Abstract. We consider a nonlinear network of three identical cells coupled in such a manner that there are multiple signal transmission time delays. In particular, such time delays are modelled as discrete and distributed delays. We present some preliminary results on the effect of distributed delay on the stability boundary of the trivial equilibrium. We also discuss the possibility of an equivariant Hopf bifurcation of this equilibrium.

Keywords. delay differential equations, distributed delay, equivariant Hopf bifurcation, stability.

AMS (MOS) subject classification: 34K15, 58F36, 92C20.

1 Introduction

In this article, we study a nonlinear network of three identical artificial neurons, which are coupled and modelled so that there are inherent multiple signal transmission delays. Such a network is modelled by the following coupled nonlinear system of delay differential equations (DDEs)

$$\dot{x}_j(t) = -x_j(t) + \alpha \int_{-\infty}^{t} k(t-s) f(x_j(s)) ds + \beta[f(x_{j-1}(t-\tau_n)) + f(x_{j+1}(t-\tau_n))] \ , \quad (1)$$

where j mod 3, α and β are real numbers, $k(\cdot)$ is an arbitrary distribution, $f : \mathbb{R} \to \mathbb{R}$ is a nonlinearity satisfying certain axioms to be specified below, and $\tau_n > 0$ is a discrete time delay. The system (1) models a network of three identical neurons, coupled in such a way that the resulting network experiences signal transmission time delays in its nearest-neighbour as well as in its self connections. The nearest-neighbour time delay is taken to be discrete, and is denoted by $\tau_n \geq 0$. On the other hand, the signal transmission time delay in the network self connections is assumed to be distributed, with kernel $k(\cdot)$. The real parameters α and β denote the self- and nearest-neighbour connections strengths, respectively. This type of network configuration has been the subject of many previous studies in the research literature (see [11] and references therein.). Whereas many such previous studies have assumed that all the network signal transmission time delays are discrete, we have here introduced a distributed signal transmission time delay in the self connections.